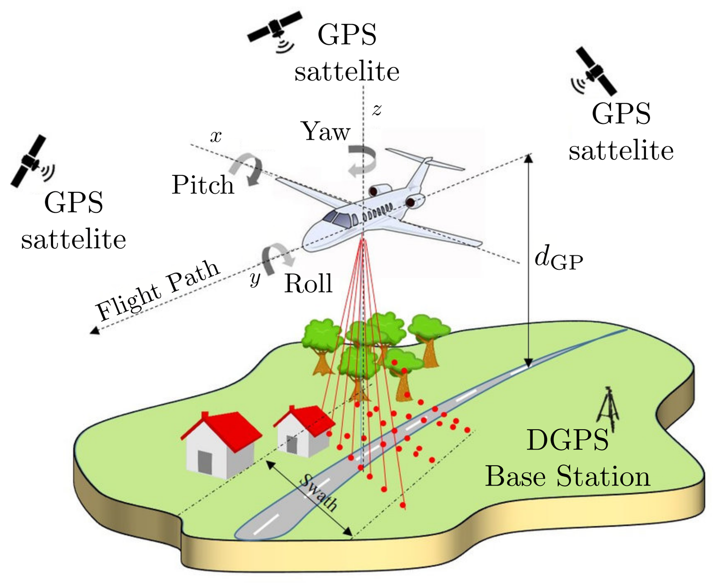
Department of Mathematics and Descriptive Geometry

# CORRECTION OF WATERCOURSES IN MAPS USING AIRBORNE LASER SCANNING DATA

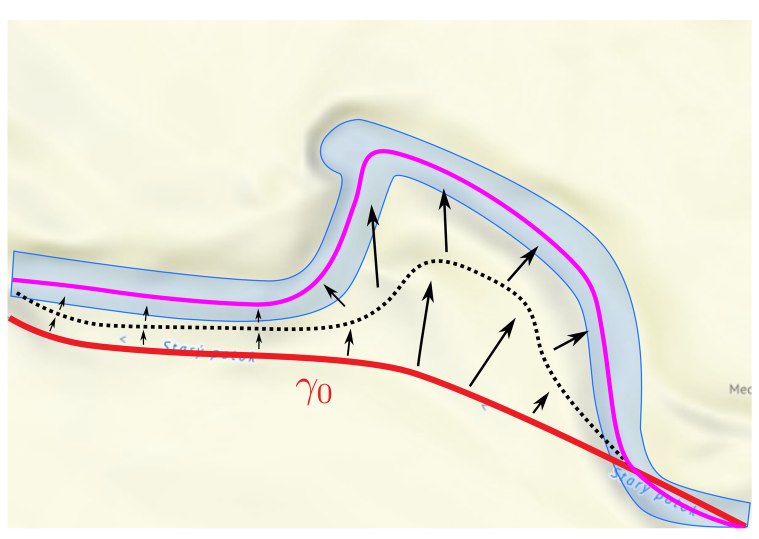
Tomáš Homola

## Motivation

Watercourses are often plotted incorrectly on maps. However, thanks to the new Digital Terrain Model, created as part of the project titled "Airborne Laser Scanning (ALS) and DTM 5.0" launched by the ÚGKK SR, we can manually fix the inaccurate mapping of watercourses. This is possible thanks to the high resolution (1m×1m) of the digital terrain model, meaning, we can see where the real waterbed is located. Thus, we can contribute ourselves to having much more accurate maps for everyone. The scanning is done by using an aircraft attached with a LiDAR sensor, together with a GPS device and a computer processing all the data.

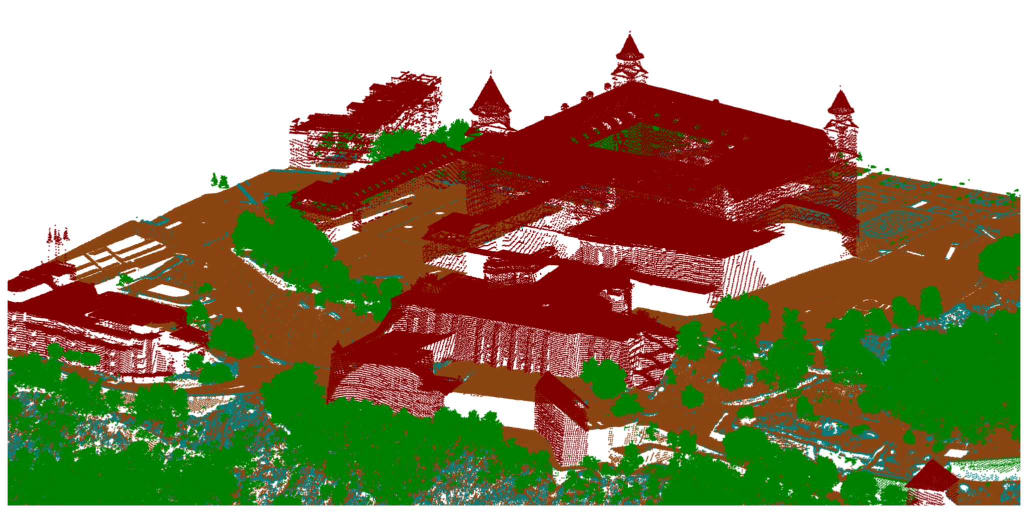



The basic approach we propose is to take the current mapping $\gamma_0$ as the initial condition for curve evolution and use advection by properly designed velocity vector field $\vec{V}$ to guide $\gamma_0$ into the waterbed, as shown in the image bellow.



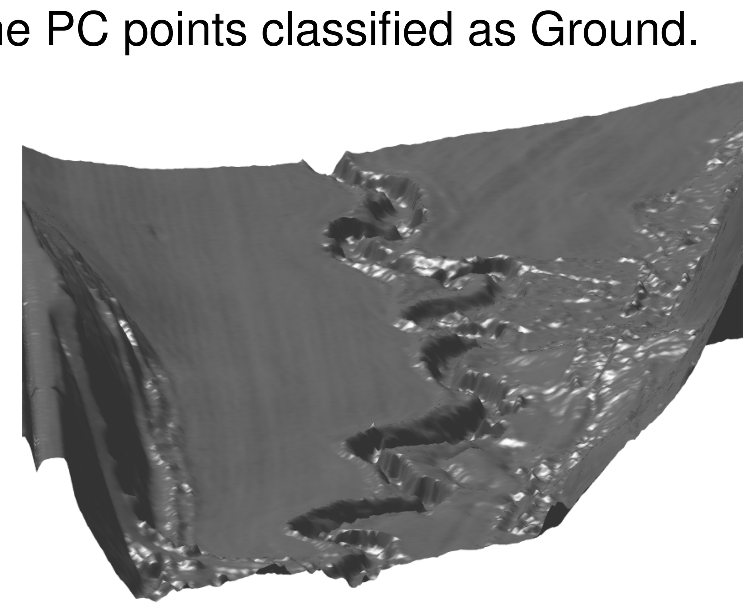

## Classified Point Cloud

Result of the ALS is a so called Point Cloud (PC) - set of points described by their GPS coordinates and other attributes, such as intensity or color of the reflected light pulse. The scanned PC is then classified into several classes. In the image bellow there is a part of classified PC around Bratislava castle. Different colors signify different classes of points. However, we will focus only on the points classified as *Water* - simply called water points.



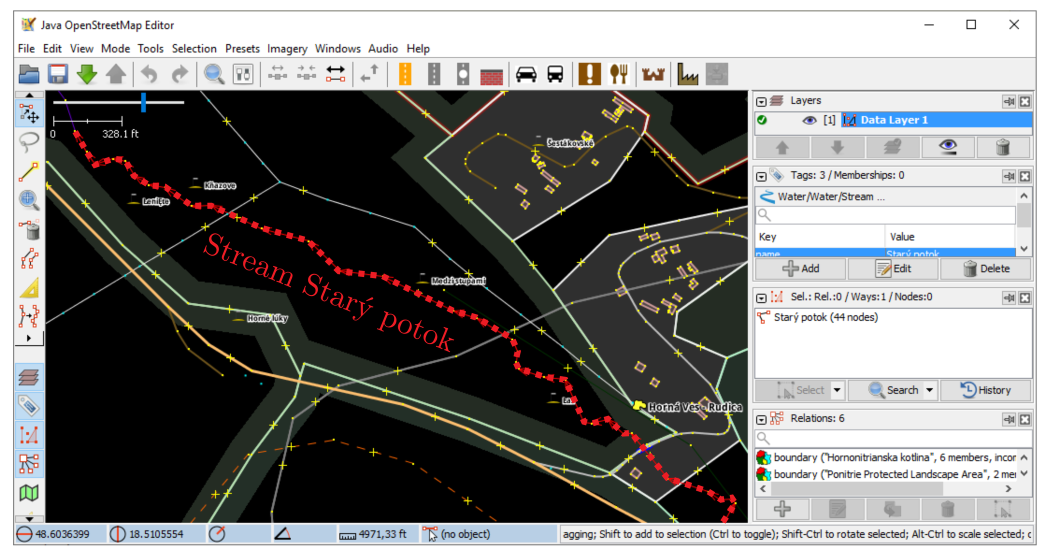
## Digital Terrain Model

Or DTM, is a digital representation of a specified reference terrain, approximated by a set of discrete terrain points arranged in a uniform grid where each point is prescribed with a single value of the terrain elevation. DTM is calculated by interpolation over the PC points classified as Ground.



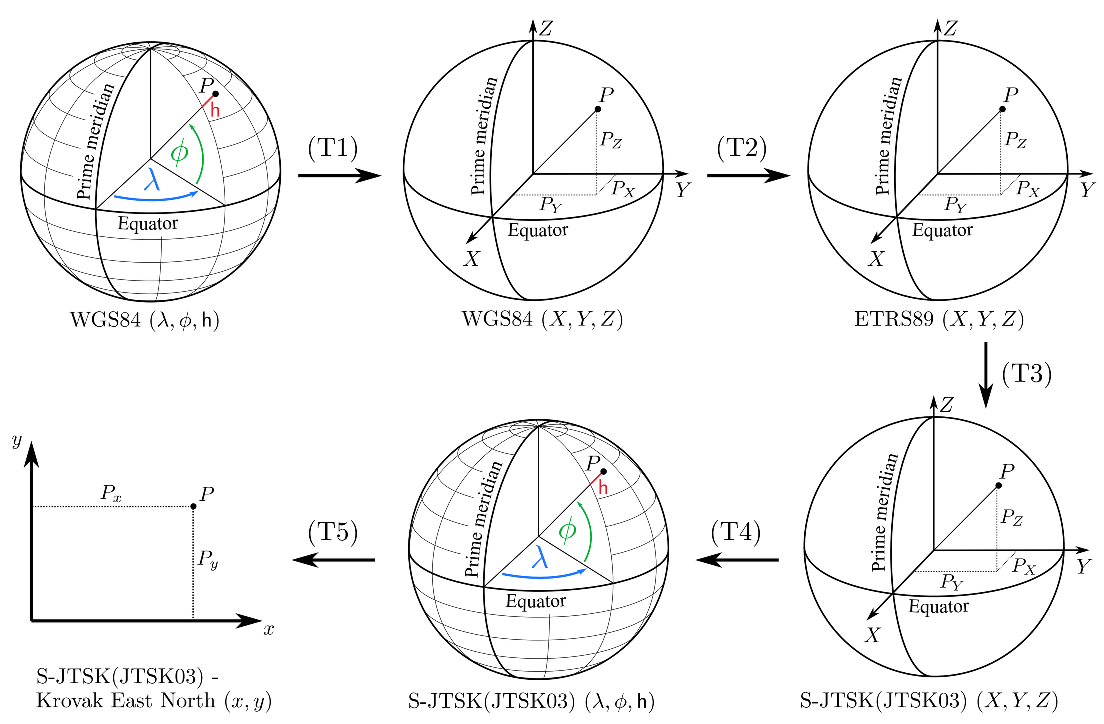
## OpenStreetMap data

Watercourses are represented on maps by a piecewise linear curve defined by a sequence of its nodal points using the GPS coordinates. These coordinates can be downloaded from OSM database using free map editor JOSM.




## Transformation of coordinates

Coordinates of the watercourse's nodal points are in different coordinate system. Therefore, we must convert them from geodetic WGS84 coordinates to S-JTSK(JTSK03) - Krovak East North grid coordinates, before we can use them with the terrain data.

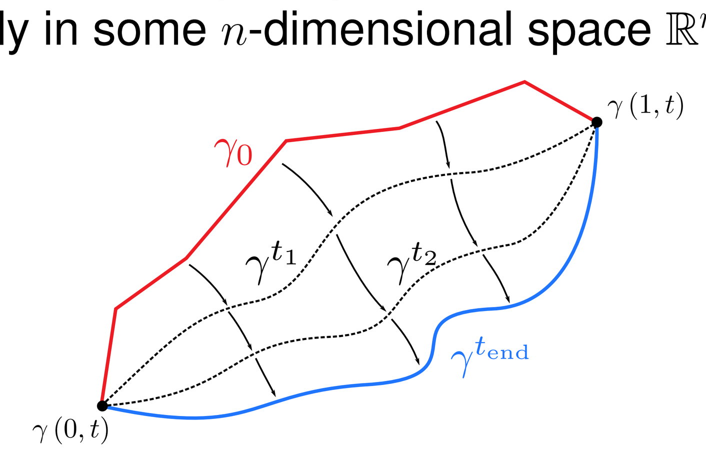



## Evolving parametric curve

As the backbone of our mathematical model will be considered the *evolving parametric curve* $\gamma$, formally defined as a map

$$\gamma\colon U \times \mathrm{T} \to \mathbb{R}^2\colon\ (u,t) \mapsto \gamma(u,t)\,,$$

that for each value of parameter $u \in U = [0,1] \subset \mathbb{R}$ at time $t \in \mathrm{T} = [0, t_{\text{end}}]$ assigns a point in $\mathbb{R}^2$ (or generally in some $n$-dimensional space $\mathbb{R}^n$).




Furthermore, we will also define the *unit tangent vector*

$$T(u,t) = \frac{\partial_u \gamma(u,t)}{\|\partial_u \gamma(u,t)\|}$$

and *positively oriented unit normal vector* $N^{+}(u,t)$, which is simply $T(u,t)$ rotated 90 degrees anticlockwise.

## Mathematical model

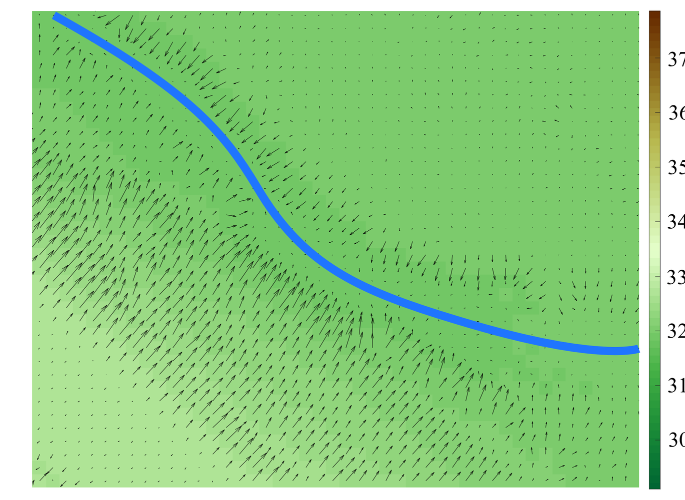
The very basic idea behind the model we propose is to prescribe to each point $\gamma(u,t)$ a velocity vector $\vec{v}(u,t)$ which will describe the direction in which it should move. We can express it by the following PDE

$$\partial_t \gamma(u,t) = \vec{v}(u,t)$$

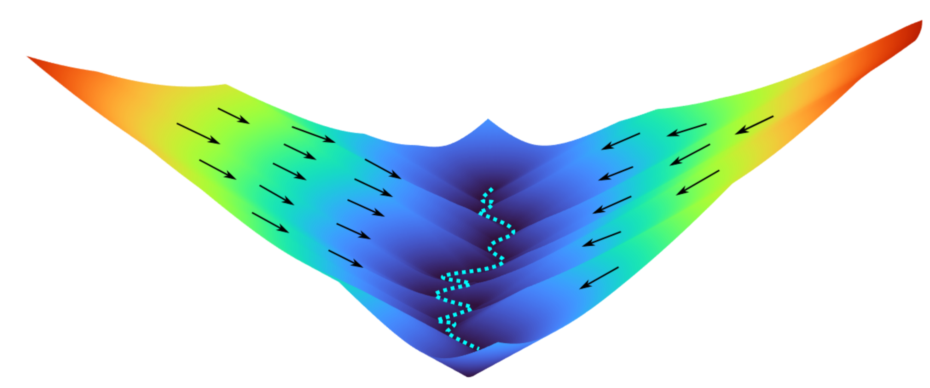
where $u \in\ ]0,1[,\ t \in\ ]0, t_{end}]$ and which is subject to the initial condition $\gamma(u,0) = \gamma_0(u)$, together with the Dirichlet boundary condition $\gamma(0,t) = G_0$, $\gamma(1,t) = G_1$ where $G_0, G_1 \in \mathbb{R}^2$, meaning that endpoints of the curve $\gamma$ will be fixed during the evolution. Next we rewrite it into

$$\partial_t \gamma = \beta N^{+} + \alpha T,$$

where the function $\beta$ is called the *normal speed* and $\alpha$ is called the *tangential speed*. For the normal speed $\beta$ we will use the negative gradient $-\nabla h$ of the terrain function $h$, see image bellow.




Also, we will use the negative gradient $-\nabla d$ of the distance function $d$ to the waterpoints. 3D plot of the distance function $d$:



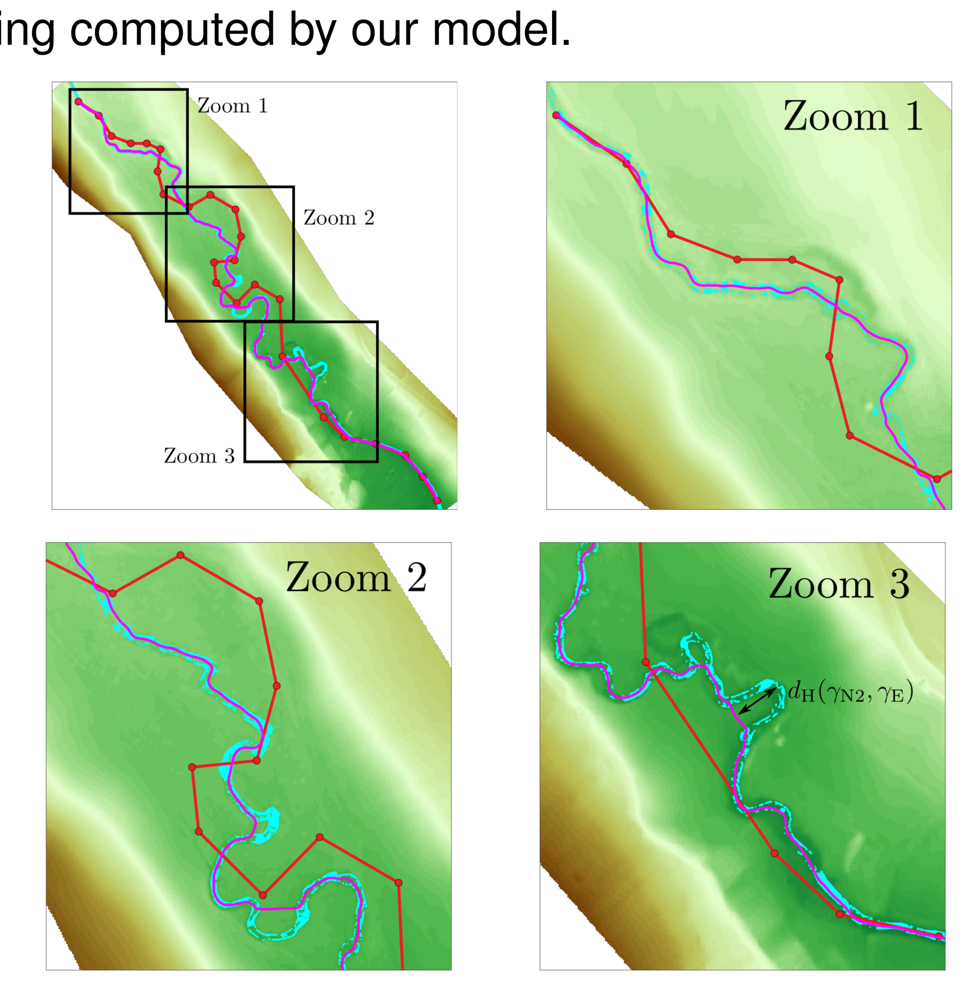
We will use weighted combination of the two negative gradients $-\nabla h$, $-\nabla d$ which will be projected onto $N^{+}$, together the with curvature regularization term $\kappa^{\pm} N^{+}$, where $\kappa^{\pm}$ is the signed curvature. The normal speed $\beta$ will be defined as

$$\beta = \delta_{\text{ext}}\big(-(1-\theta)\,\nabla h - \theta \nabla d\big)\cdot N^{+} + \delta_\kappa \kappa^{\pm}$$

where $\theta$ controls the contribution of each gradient and $\delta_{\text{ext}}$ and $\delta_\kappa$ are parameters controlling strength of each term. For the tangential speed $\alpha$ we used the so called asymptotically uniform redistribution of points.

## Numerical experiments

Results of the numerical experiment performed with stream Parná. By visual comparison: in red is the original mapping, in magenta is the new mapping computed by our model.




We can see, that the majority of the new mapping is much more accurate compared to the original one. Comparison using Hausdorff distance $d_{\mathrm{H}}$ and average Hausdorff distance $\overline{d}_{\mathrm{H}}$:

| | |
|---|---|
| $d_{\mathrm{H}}(\gamma_0, \gamma_{\mathrm{E}})$ | 64.69 m |
| $\overline{d}_{\mathrm{H}}(\gamma_0, \gamma_{\mathrm{E}})$ | 18.06 m |

| | |
|---|---|
| $d_{\mathrm{H}}(\gamma_{\mathrm{N2}}, \gamma_{\mathrm{E}})$ | 34.79 m |
| $\overline{d}_{\mathrm{H}}(\gamma_{\mathrm{N2}}, \gamma_{\mathrm{E}})$ | 1.86 m |

where $\gamma_0$ is the original mapping, $\gamma_{\mathrm{E}}$ represents the *ground truth* and $\gamma_{\mathrm{N2}}$ is the new mapping. Average error of the new mapping is close to the resolution of the DTM (1m).

**To see more numerical experiments, use the following QR code:**

SCAN ME

Supervisor: Ing. Mgr. Lukáš Tomek, PhD.
Student: Tomáš Homola
Academic year: 2021/2022
Study field: 9.1.9. Applied Mathematics
BACHELOR'S THESIS
Correction of watercourses in maps using airborne laser scanning data