Set Difference and Symmetric Difference of Fuzzy Sets

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Outline

Classical set theory

Set operations

- Union- $\cup$
- Intersection - $\cap$
- Complement - ^{c}
- Difference -$\setminus$
- Symmetric difference - Δ
- ....

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Classical vs Fuzzy operators

Set operations

Operation	Classical	Fuzzy
Union	$\cup$	t-conorm
Intersection	$\cap$	t-norm
Complement	^{c}	Fuzzy Negation
Set difference	$\setminus$	??
Symmetric difference	$\triangle$	??

Table : Classical vs Fuzzy operators

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Difference of (classical)sets

Set difference

The set difference of A, B is defined as

$$A \setminus B = \{a \in A | a \notin B\}$$

Equivalent definition

$$A \setminus B = A \cap B^c$$

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Generalisations of Difference operators

Definition

The difference of two fuzzy sets can be defined as

$$S_{d_1}(x, y) = T(x, N(y)) \quad (1)$$

$$S_{d_2}(x, y) = x - T(x, y) \quad (2)$$

where T, N are a t-norm and a fuzzy negation, resp.

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Outline

Dubois and Prade (1980)

Two Examples

- $S_1(x, y) = |x - y|$
- $S_2(x, y) = \max(\min(x, 1 - y), \min(1 - x, y))$

Note

- S_1, S_2 are only examples of fXoR operators
- No axiomatic definition was proposed.

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Alsina et.al. (2005)

Definition

$\Delta : [0,1]^2 \longrightarrow [0,1]$ is called a symmetric difference operator if

(A1) $\Delta(a,0) = \Delta(0,a) = a$

(A2) $\Delta(a,a) = 0$

(A3) $\Delta(a,1) = \Delta(1,a) = N(a)$, where N is a strong negation.

Drawbacks

- Δ is not commutative
- N is a strong negation
- Not general enough to accomodate known fXoR operators.
- $S_2(a,a) \neq 0$.

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- Most of the fXoR operators do not satisfy associativity
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fXoR operators - some Generalisations

Classical $A \Delta B$	fXoR
$(A \setminus B) \cup (B \setminus A)$	$D_1(x,y) = S(x - T(x,y), y - T(x,y))$
$(A \cap B^c) \cup (B \cap A^c)$	$D_2(x,y) = S(T(x, N(y)), T(y, N(x)))$
$(A \cup B) \cap (A \cap B)^c$	$D_3(x,y) = T(S(x,y), N(T(x,y)))$
$(A \cup B) \cap (A^c \cup B^c)$	$D_4(x,y) = T(S(x,y), S(N(x), N(y)))$
$(A \cup B) \setminus (A \cap B)$	$D_5(x,y) = S(x,y) - T(S(x,y), T(x,y))$

Table : Various fXoR operators

fXoR operators - some Generalisations

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Table : Various fXoR operators

Properties of D_i

D_i and the violated axioms

	Alsina .*et.al*	Bedregal.*et.al.*
D_1	$A2$	$B2$
D_2	$A2, A3$	$B2$
D_3	$A2, A3$	$B2$
D_4	$A2, A3$	$B2$
D_5	$A2$	$B2$

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   - Introduction
   - Earlier work
2. Symmetric Difference
   - Definition
   - Examples
   - Properties
   - Applications
   - Future Work
   - References

Definition 1

$D : [0,1]^2 \to [0,1]$ is called an **fXoR** operator if for all x, $y \in [0,1]$ it satisfies:

(i) $D(x,y) = D(y,x)$,
(ii) $D(0,x) = x$,
(iii) $D(1,x) = N(x)$ where N is a fuzzy negation.

Theorem

D_i satisfies Definition 1, for $i = 1, 2, 3, 4, 5$.

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Figures

Case 1: Consider $T = \min$, $S = \max$ and $N(x) = 1 - x$, then $D_1 - D_5$ are shown in Fig. 1.
Case 2: when $T = T_{\mathbf{LK}}$, $S = S_{\mathbf{P}}$ and $N(x) = 1 - x$, then $D_1 - D_5$ are shown in Fig. 2.

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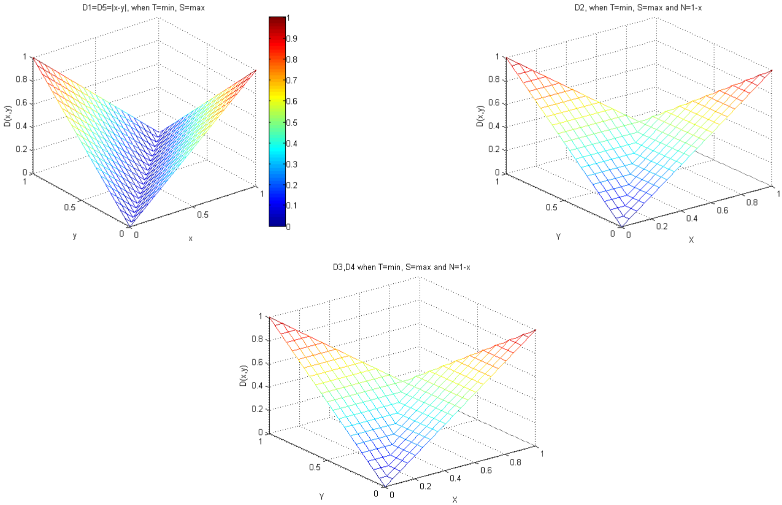

Figure : (a) D_1, D_5 (b) D_2 (c) D_3, D_4, when $T = T_M$, $S = S_M$ and

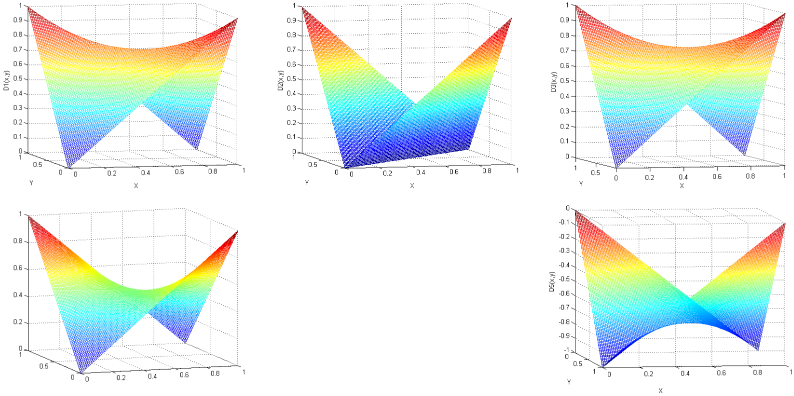

Figure : (a) D_1 (b) D_2 (c) D_3 (d) D_4 (e) D_5, when $T = T_{\mathbf{LK}}$, $S = S_{\mathbf{P}}$ and $N(x) = 1 - x$

Outline

Properties of fXoR operators

Classical $A \Delta B$	fXoR
$A \Delta (B \Delta C) = (A \Delta B) \Delta C$	$D(x, D(y, z)) = D(D(x, y), z)$
$A \Delta A = \emptyset$	$D(x, x) = 0$
$A \Delta B = A^c \Delta B^c$	$D(x, y) = D(N(x), N(y))$
$(X \Delta A)^c = X \Delta A$	$D(1, y)$ is strong negation
$A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$	$T(x, D(y, z)) = D(T(x, y), T(x, z))$

Table : Various operators

Properties of fXoR operators

Classical $A \Delta B$	**fXoR**
$A\Delta(B\Delta C) = (A\Delta B)\Delta C$	$D(x, D(y,z)) = D(D(x,y),z)$
$A\Delta A = \emptyset$	$D(x,x) = 0$
$A\Delta B = A^c \Delta B^c$	$D(x,y) = D(N(x), N(y))$
$(X\Delta A)^c = X\Delta A$	$D(1,y)$ is strong negation
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Table : Various operators

Properties of fXOR operators

Property	D_1	D_2	D_3	D_4	D_5
Commutativity	✓	✓	✓	✓	✓
Associativity	×	×	×	×	×
$D(0, x) = x$	✓	✓	✓	✓	✓
$D(1, x) = D(x, 1)$	$1 - x$	$N(x)$	$N(x)$	$N(x)$	$1 - x$
$D(N(x), N(y)) = D(x, y)$	×	×	×	×	×
$D(x, x) = 0$	×	×	×	×	×
Distributivity w.r.t T	×	×	×	×	×
$D(x, y) = 1 \Rightarrow \|x - y\| = 1$	×	×	×	×	×
Conditional Monotonicity	✓	×	✓	×	×

Table : Properties vs Various operators

Properties of fXOR operators

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Associativity	×	×	×	×	×
$D(0, x) = x$	✓	✓	✓	✓	✓
$D(1, x) = D(x, 1)$	$1 - x$	$N(x)$	$N(x)$	$N(x)$	$1 - x$
$D(N(x), N(y)) = D(x, y)$	×	×	×	×	×
$D(x, x) = 0$	×	×	×	×	×
Distributivity w.r.t T	×	×	×	×	×
$D(x, y) = 1 \Rightarrow \|x - y\| = 1$	×	×	×	×	×
Conditional Monotonicity	✓	×	✓	×	×

Table : Properties vs Various operators

More Properties

Definition

Given $x, y, z \in [0, 1]$, fXoR operator D is said to satisfy

(i) (CP) Cancellative Property if $D(x, y) = D(x, z)$ then $y = z$.

(ii) (EP) Exchange Principle if $D(x, y) = z$ then $D(y, z) = x$ and $D(x, z) = y$.

(iii) (COP) Coincidence Principle if $D(x, y) = 0 \Leftrightarrow x = y$.

(iv) (DT) Delta transitivity if $D(D(x, y), D(y, z)) = D(x, z)$.

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Few relationships

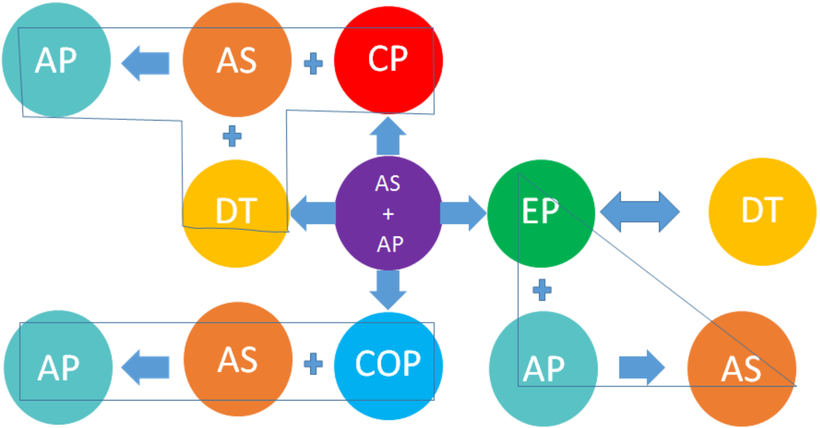

Figure : Some of the relationships between the different properties

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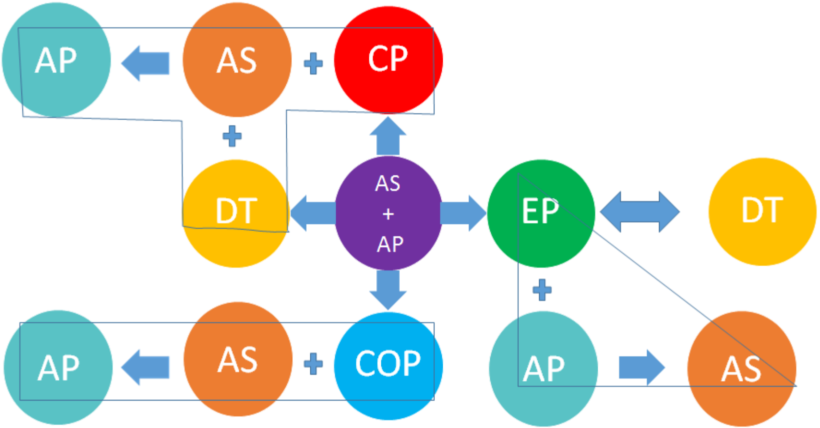

Figure : Some of the relationships between the different properties

Outline

Applications

- Fuzzy XoR function is constructed and is used to extract edges from grayscale images [1].
- In [2], Pedrycz et.al., developed a logic-based architecture of fuzzy neural networks, called here fXoR networks using basic fuzzy operations such as Fuzzy Negation, t-norm and t-conorm.
- Commonly used preference formation rules in psychology and marketing using linear models is given in [3]. The interaction term with the linear models includes counterbalance (fXoR).

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- Properties of $x + y - 2xy$ is studied in [4] and it is found that the operation is least sensitive (Most Robust) on average.
- Fuzzy XoR dataset was used to compare the performance of Fuzzy Clustering lagorithms namely Fuzzy C-Means (FCM), Gustafson-Kessel FCM, and Kernel-based FCM in [5].
- Classical connective XoR is frequently used as a problem, as, e.g., in Neural Networks, in support vector machines (SVM) and Quantum Computing due to its non-linearity.

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Future Work

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- All the properties of families in detail
- Intersection between the families
- Characterization of different families

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Thank you!!!

Questions???

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