High-resolution modelling of altimetry-derived gravity data by solving the altimetry-gravimetry boundary-value problem

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Abstract

We present a numerical solution of the altimetry-gravimetry boundary-value problem using the finite volume method (FVM). Such a numerical approach discretizes the 3D computational domain between an ellipsoidal approximation of the Earth's surface and an upper boundary at the mean altitude of the GOCE orbits. A parallel implementation of FVM and large-scale parallel computations on clusters with distributed memory allow high-resolution modelling of the altimetry-derived gravity disturbances over oceans. The key idea is to generate precisely the Dirichlet boundary conditions over oceans that are prescribed in form of the disturbing potential. For this purpose the geopotential on the mean sea surface is generated from the GRACE/GOCE-based satellite-only geopotential model and afterwards filtered by nonlinear diffusion filtering. On the upper boundary, the Dirichlet boundary conditions generated from the same satellite-only geopotential model are prescribed as well. Numerical experiments present high-resolution modelling of the gravity disturbances over oceans derived from the DTU13 mean sea surface model and the GO_CONS_GCF_2_DIR_R5 geopotential model up to degree 300. Their comparison with the DTU13_GRAV altimetry-derived gravity data indicates an importance of a-priori information about the mean dynamic topography, especially in zones of main ocean geostrophic surface currents.

Satellite-fixed altimetry-gravimetry BVP

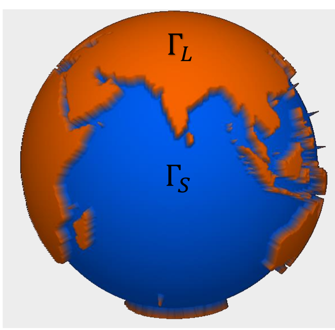
We consider the following altimetry-gravimetry boundary-value problem (AGBVP) in the bounded domain Ω:

$$\Delta T = 0 \ in \ \Omega, \qquad (1)$$

$$T = T^{MSS} on \ \Gamma_S, \qquad (2)$$

$$\langle \nabla T, \vec{s} \rangle = \delta g \ on \ \Gamma_L, \qquad (3)$$

$$T = T_{SAT} \quad on \ \Gamma_{SAT}, \qquad (4)$$



where Ω is the exterior space above the Earth, T is the disturbing potential, Γ_S is the area of sea on the bottom boundary of Ω, Γ_L is the area of land on the bottom boundary of Ω and Γ_{SAT} is the boundary of Ω-Γ_S-Γ_L.

The Finite Volume Method for the AGBVP

To solve (1)-(4), the finite volume method (FVM) has been chosen. In the FVM, we divide the domain *Ω* into finite volumes *p*, multiply the Laplace equation (1) by minus one and integrate the resulting equation over each finite volume with a use of the divergence theorem.
Let us denote $q \in N(p)$ as a neighbor of the finite volume *p* and *N(p)* denotes all neighbors of *p*. Let T_p and T_q be approximate values of *T* in *p* and *q*, e_{pq} be a boundary of the finite volume *p* common with *q*, $\vec{n}_{pq}$ be its unit normal vector oriented from *p* to *q*, $m(p)$ is the volume of *p* and $m(e_{pq})$ is the area of e_{pq}. Let x_p and x_q be representative points of *p* and *q* respectively and d_{pq} their distance. If we approximate the normal derivative along the boundary of the finite volume *p* by differences between T_p and T_q divided by their distance d_{pq}, we obtain the following equation for every finite volume *p*

$$\sum_{q \in N(p)} m(e_{pq}) \frac{(T_p - T_q)}{d_{pq}} = 0, \qquad (5)$$

which forms the linear system of algebraic equations. The term $\frac{m(e_{pq})}{d_{pq}}$ defined on sides of the finite volume *p* is referred to as the transmisivity coefficient. The system (5) must be accompanied by the boundary conditions. For volumes along boundaries, case of the Dirichlet BCs (2,4), we prescribe the disturbing potential for T_q in (5), and move the term $m(e_{pq})T_q/d_{pq}$ to the right-hand side. For the Neumann BC applied on the bottom boundary, we prescribe δg for value $(T_p - T_q)/d_{pq}$ in (5), and move $m(e_{pq})\delta g$to the right-hand side and update correspondingly the diagonal coefficient.

Altimetry-derived gravity disturbances

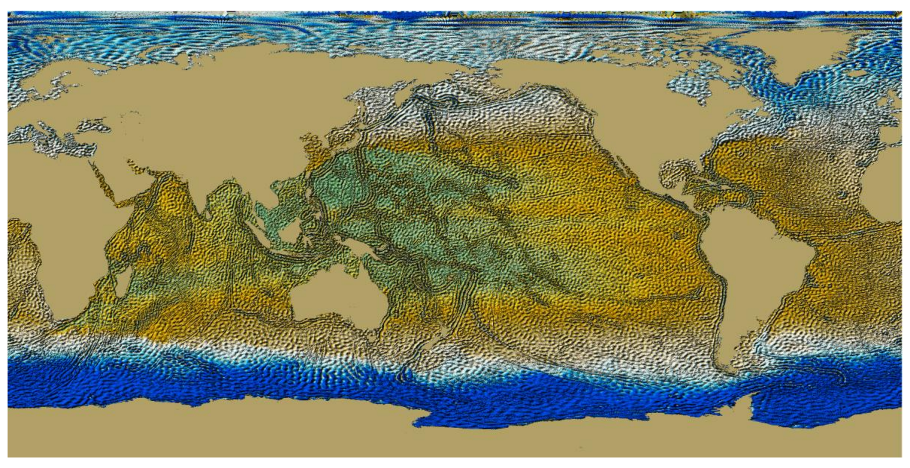
By solving the Satellite-fixed AGBVP in a space domain we obtain solution in every discrete point T_p. To approximate a derivative is possible to use the finite difference forward scheme in form[1]:

$$\frac{-\frac{137}{60}T_p + 5T_{q_{+1}} - 5T_{q_{+2}} + \frac{10}{3}T_{q_{+3}} - \frac{5}{4}T_{q_{+4}} + \frac{1}{5}T_{q_{+5}}}{d_{pq}} \approx \delta g, \qquad (6)$$

where $T_{q_{+1}}$ is neighbor T_p in radial direction.

[1]*Fornberg, Bengt (1988), "Generation of Finite Difference Formulas on Arbitrarily Spaced Grids", Mathematics of Computation* 51 (184): 699– 706, *ISSN* 0025 – 5718.

Nonlinear diffusion filtering of the geopotential



Picture 1: Geopotential.

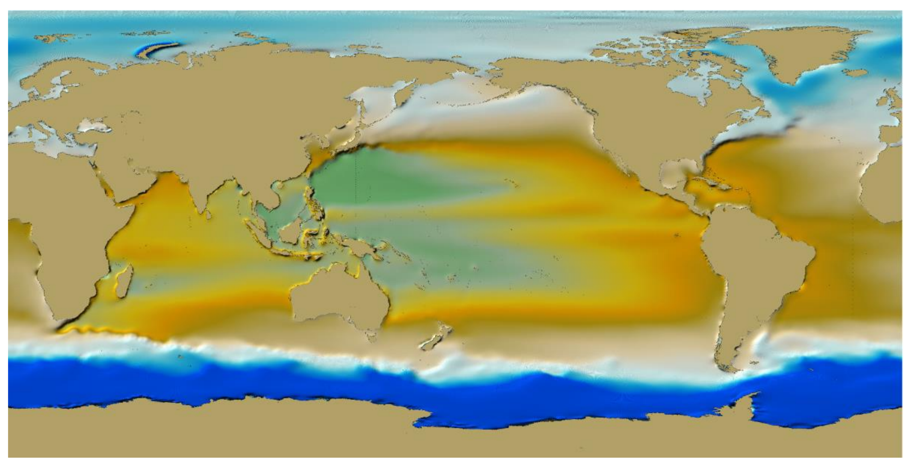
Input data:

Geopotential generated from GO_CONS_GCF_2_DIR_R5 on DTU13_MSS

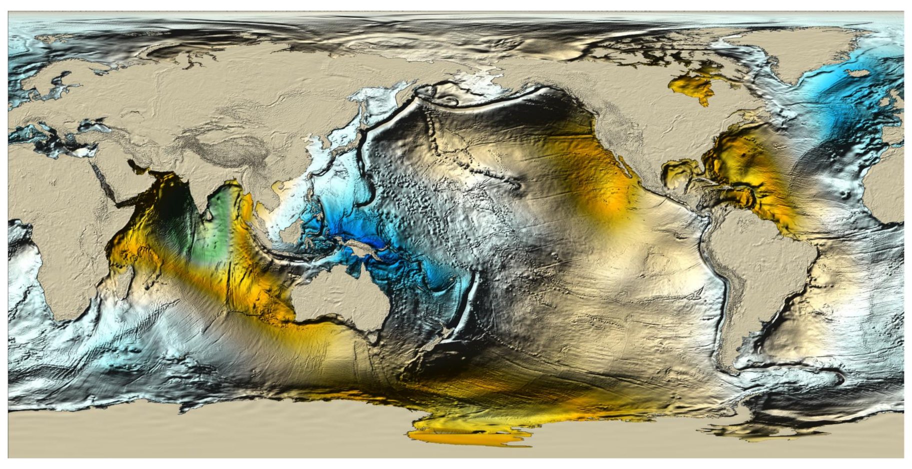
Regularized Perona-Malik model

$$\frac{\partial W}{\partial t} - \nabla_s \cdot (g|\nabla_s W^\sigma|\nabla_s W) = 0$$

where ∇_s represents a surface gradient, g is the edge detector defined as $g|\nabla_s W^\sigma| = {}^{1}/_{1+H|\nabla_s W^\sigma|^2}$ and $W^\sigma$is the solution of the linear diffusion obtained for a short time step[2].



Picture 2: Filtered geopotential.



Picture 3: Disturbing potential obtained from filtered geopotential.

Output data:

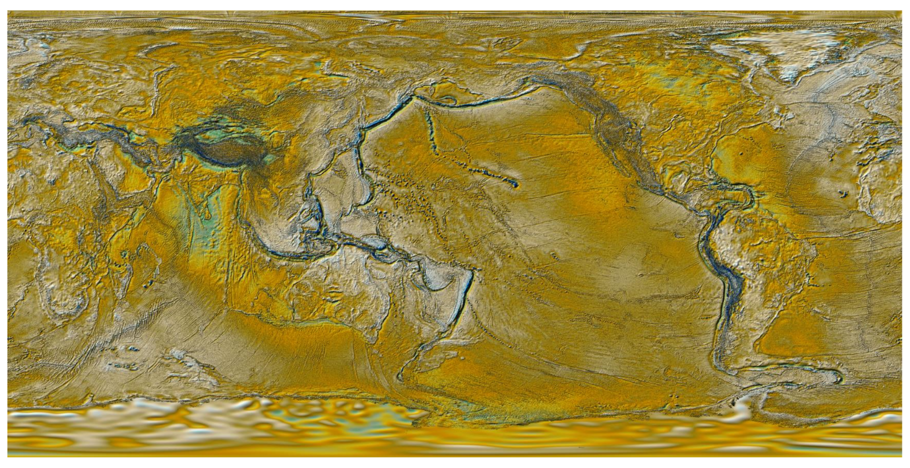
Originally filtered disturbing potential

[2]R.Cunderlik, K.Mikula, M.Tunega, Nonlinear diffusion filtering of data on the Earth's surface, Journal of Geodesy, Vol. 87 (2013) pp. 143–160.

High-performance computing

About experiment:

Resolution: 2' x 2' x 400 m
(10 800 x 5 400 x 250)
Altitude of the upper boundary: 100 km
Memory requirements: 870 GB
Computational time: ≈ 24h on 128 CPU



Picture 4: Altimetry-derived gravity disturbances [mGal].

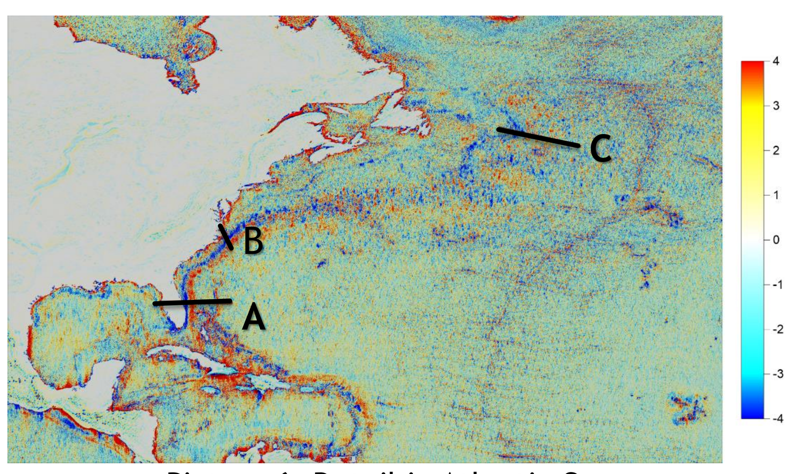
Input data:

Upper boundary:
- Disturbing potential: GO_CONS_GCF_2_DIR_R5 (SH up to d/o 300)

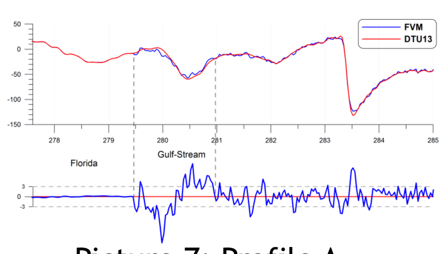
Bottom boundary:
- Gravity disturbances on land: DTU13_GRAV
- Disturbing potential on oceans: generated from GO_CONS_GCF_2_DIR_R5 on DTU13_MSS and filtered using nonlinear diffusion filtering

Comparison with DTU13_GRAV

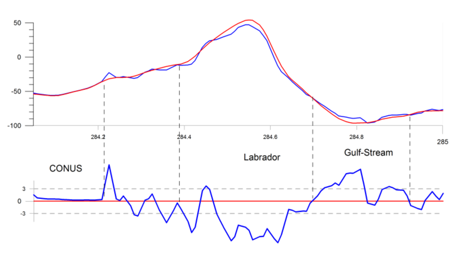
Detail in Atlantic Ocean (Gulf Stream)



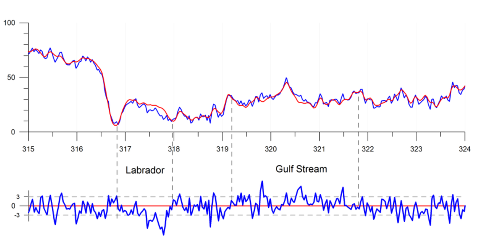
Picture 6: Detail in Atlantic Ocean.



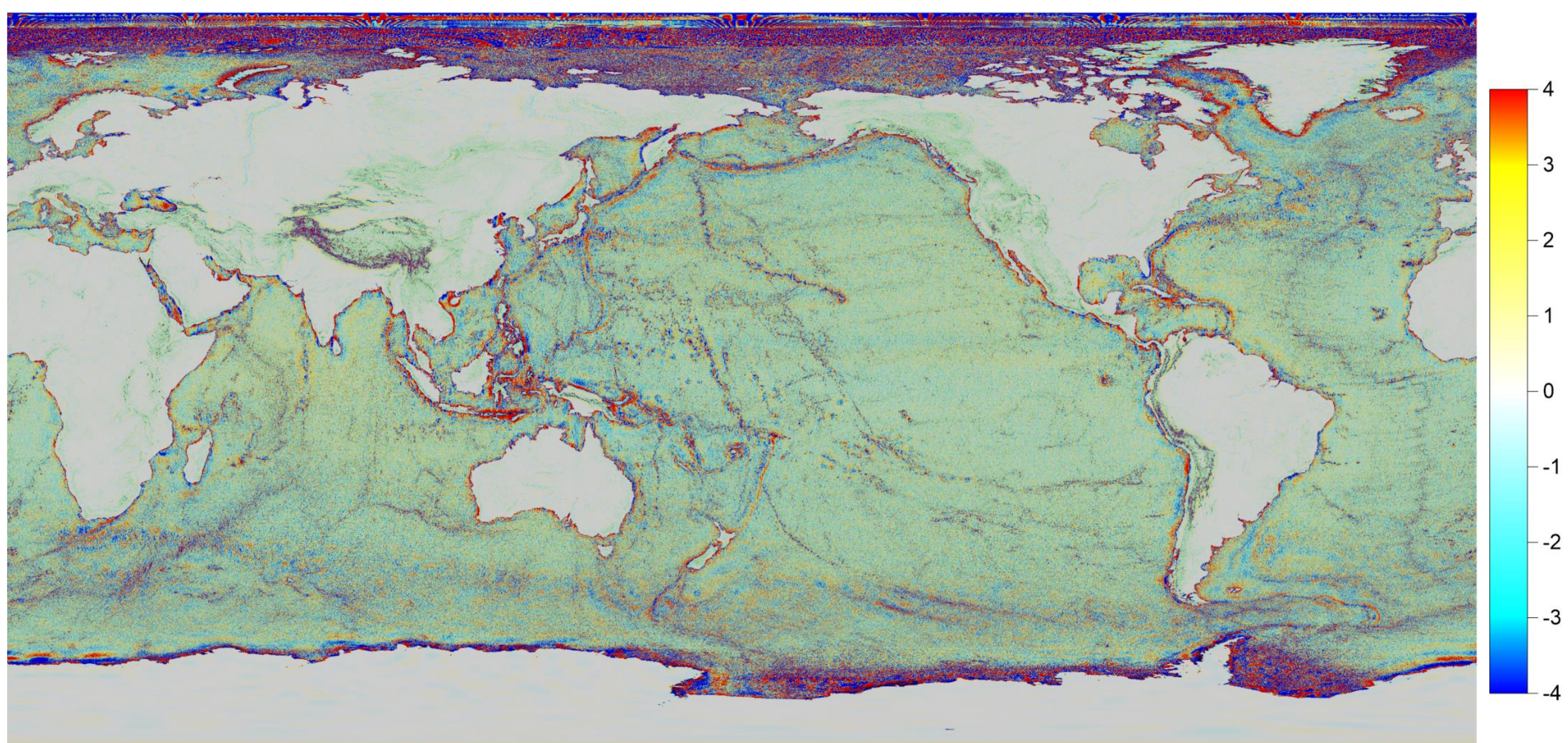
Picture 7: Profile A.



Picture 8: Profile B.



Picture 9: Profile C.

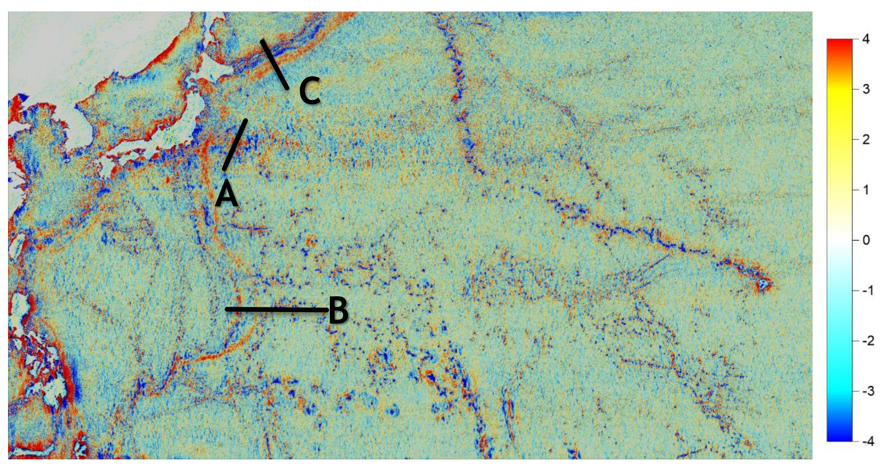


Picture 5: Statistics of residuals [mGal] between our solution and DTU13_GRAV.

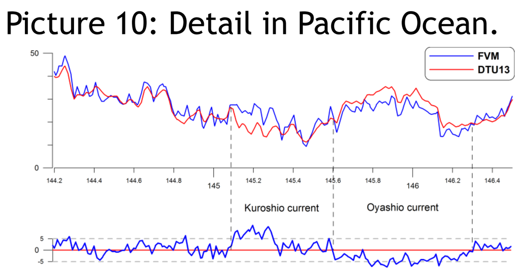
Table 1: Statistics of residuals [mGal] between our solution and DTU13_GRAV.

Oceans	TOTAL	Pacific	Atlantic	Indian	Arctic	Southern
Mean	0.028	-0.004	-0.018	0.090	0.218	-0.053
STD	4.303	2.996	3.299	4.202	6.944	5.479

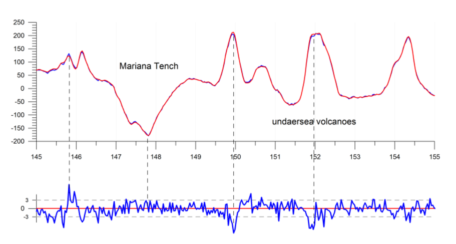
Detail in Pacific Ocean (Kuroshio)



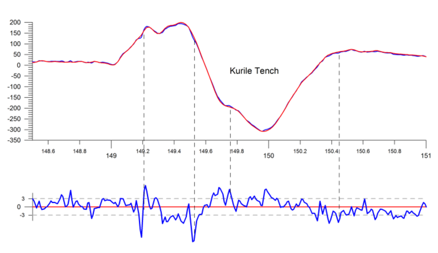
Picture 10: Detail in Pacific Ocean.



Picture 11: Profile A.



Picture 12: Profile B.



Picture 13: Profile C.

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