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Determination of W0 from the GOCE measurements using the method of fundamental solutions

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Determination of W_0 from the GOCE measurements using the method of fundamental solutions

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Abstract: The paper presents the method of fundamental solutions (MFS) applied for global gravity field modelling. MFS as an inherent mesh-free method is used to derive the geopotential and its first derivatives from the second derivatives observed by the GOCE satellite mission, namely from the radial components of the gravity tensor. Unknown coefficients of the approximate solution by MFS are determined at the source points located directly on the Earth's surface. Afterwards, the disturbing potential or gravity disturbance can be evaluated at any point above the Earth's surface. To get their values on the Earth's surface, singularities of the fundamental solutions need to be overcome. In this paper two strategies are used: (i) the source points are located on the fictitious boundary, which is situated below the Earth's surface, or (ii) ideas of the singular boundary method that isolate the singularities are implemented. The paper studies how a depth of the fictitious boundary influences accuracy of the MFS solutions. All particular solutions are compared with the GOCO03S satellite-only geopotential model and the EGM-2008 combined model. Finally, the geopotential on the DTU10 mean sea surface is evaluated from the MFS solutions resulting in the W0 estimates.

Key-words: *Method of fundamental solutions, global gravity field modelling, GOCE measurements, geopotential on the mean sea surface, W_0 estimates*

1 Introduction

A unification of local vertical datums and establishment of the World Height System (WHS) is one of the main tasks of modern geodesy. It involves a determination of W_0 as a reference value of the geopotential on the geoid. All recent W_0 estimates are basically derived from the global geopotential models (GGMs) that are developed using the spherical harmonics (SH) approach, cf. (Burša et al. 2007), (Sánchez 2009), (Dayoub et al. 2012), (Čunderlík et al. 2014). However, the GOCE satellite mission, which is directly measuring the second derivatives of the geopotential, has brought new opportunities in applications of other numerical approaches for global gravity field modelling. In this paper the method of fundamental solutions (MFS) is presented to derive the geopotential and its first derivatives on or above the Earth’s surface from the second derivatives observed by GOCE.

The ideas behind MFS were primarily developed by V. D. Kupradze and M. A. Alexidze in the late 1950s and early 1960s (Kupradze and Alexidze 1964). However, MFS as a computational technique was proposed much later by R. Mathon and R. L. Johnston in the late 1970s (Mathon and Johnston 1977). In the 1990s, M. A. Golberg and C. S. Chen extended MFS to deal with inhomogeneous equations and time-dependent problems (Golberg and Chen 1994). Recent developments indicates that MFS has become a useful tool for solving large variety of physical and engineering problems, cf. (Hon and Wei 2005), (Fan et al. 2009) or (Chen et al. 2011). To cure the problem of a fictitious boundary in MFS, some new techniques have recently been developed, e.g. the singular boundary method (SBM) (Chen and Wang 2010).

In this paper MFS is applied to derive the disturbing potential and its first derivatives from the radial components T_{rr} of the disturbing tensor observed by GOCE. Numerical experiments show how a depth of the fictitious boundary influences accuracy of the obtained MFS solution on or above the Earth’s surface. In case that the source points are located directly on the Earth’s surface, the ideas of SBM that isolate singularities of the fundamental solution (Gu et al. 2012) are applied. Finally, the geopotential on the mean sea surface is evaluated from different MFS solutions. It allows estimating the W_0 values that can be considered independent from ones obtained from the SH-based GGMs.

2 MFS for the potential problems

MFS is a technique for the numerical solution of certain elliptic boundary value problems (BVPs) (Mathon and Johnston 1977). It belongs to the general class of the boundary collocation methods. Like the boundary element method (BEM), it is applicable when the fundamental solution of a partial differential equation (PDE) of interest is known. MFS was developed to overcome the major drawbacks of BEM, i.e., to avoid numerical integration of the singular fundamental solution by introducing a fictitious boundary (FB) outside the physical domain. In contrast to BEM, MFS is an inherent mesh-free method and does not involve integral evaluation. Hence, it provides an efficient computational alternative for problems in higher dimensions with irregular domains.

In the following we focus on the exterior potential problem in 3D that corresponds to the geodetic BVP. Let us consider the Laplace equation governing potential problems exterior a 3D domain Ω (the Earth) (Fig.1)

$$\nabla^2 u(\boldsymbol{x}) = 0, \quad \boldsymbol{x} \in ext.\Omega, \tag{1}$$

with the following boundary conditions (BC)

$$u(\boldsymbol{x}) = \bar{u}(\boldsymbol{x}), \;\; \boldsymbol{x} \in \Gamma_D \text{ (Dirichlet BC)}, \tag{2}$$

$$q(\boldsymbol{x}) = \frac{\partial u}{\partial \boldsymbol{n}}(\boldsymbol{x}) = \bar{q}(\boldsymbol{x}), \qquad \boldsymbol{x} \in \Gamma_N \text{ (Neumann BC)}, \tag{3}$$

where u is the potential field, Γ_D and Γ_N construct the whole boundary of the domain Ω and $\boldsymbol{n}$ devotes the outward normal.

An approximate solution by MFS is expressed as a linear combination of the fundamental solutions with respect to different source points

$$u(\boldsymbol{x^i}) = \sum_{j=1}^{N} \alpha_j G(\boldsymbol{x^i}, \boldsymbol{s^j}), \tag{4}$$

$$q(\boldsymbol{x^i}) = \frac{\partial u(\boldsymbol{x^i})}{\partial \boldsymbol{n}_{x^i}} = \sum_{j=1}^{N} \alpha_j \frac{\partial G(\boldsymbol{x^i}, \boldsymbol{s^j})}{\partial \boldsymbol{n}_{x^i}}, \tag{5}$$

where $\boldsymbol{x^i}$ is the i-th collocation point and $\boldsymbol{s^j}$ is the j-th source point, α_j denotes the j-th unknown coefficient of the distributed source at $\boldsymbol{s^j}$, N represents the number of source points and

$$G(\boldsymbol{x^i}, \boldsymbol{s^j}) = \frac{1}{4\pi \left| \boldsymbol{x^i} - \boldsymbol{s^j} \right|}, \tag{6}$$

is the fundamental solution of the Laplace equation in 3D, which represents the basis functions of the method. For a well-posed BVP, the unknown coefficients $\{\alpha_j\}$, j=1,…,N, can be determined by collocating N observation points with BC from Eq. (2) or (3). Once all the unknown coefficients $\{\alpha_j\}$ are solved, physical quantities at any point inside the physical domain (i.e. exterior Ω in our case) including its boundary can be easily evaluated from the field equations (4) or (5).

To avoid singularities of the fundamental solutions, the source points are located on the FB outside the computational domain. For the exterior BVP described in Eqs. (1-3), the FB is inside Ω, i.e., below the Earth's surface (Fig.1a). However, despite many years of great effort, a determination of the FB is largely based on experiences, especially for problems in complicated geometries and higher dimensions.

3 MFS for gravity field modelling from the GOCE measurements

The gravity field modelling is usually formulated in terms of the Laplace equation (1) for the disturbing potential T. The GOCE observations provide the second derivatives of the geopotential, or the disturbing potential, respectively. In this study, the radial components T_{rr} of the disturbing tensor are used to derive the unknown coefficients α_j at the source points $\boldsymbol{s^j}$ using the expression

$$T_{rr}\left(\boldsymbol{x^i}\right) = \frac{\partial^2 T\left(\boldsymbol{x^i}\right)}{\partial \boldsymbol{r}^2_{\boldsymbol{x^i}}} = \sum_{j=1}^{N} \alpha_j \frac{\partial^2 G\left(\boldsymbol{x^i}, \boldsymbol{s^j}\right)}{\partial \boldsymbol{r}^2_{\boldsymbol{x^i}}} , \qquad (7)$$

where

$$\frac{\partial^2 G\left(\boldsymbol{x^i}, \boldsymbol{s^j}\right)}{\partial \boldsymbol{r}^2_{\boldsymbol{x^i}}} = \frac{\partial^2 G_{i,j}}{\partial \boldsymbol{r_i}^2} = \frac{1}{4\pi}\left[\frac{1}{d_{ij}^3} - 3\frac{\left\langle \boldsymbol{d}_{ij}, \boldsymbol{r_i}\right\rangle^2}{d_{ij}^5}\right], \qquad (8)$$

and $\boldsymbol{r_i}$ denotes the radial vector at $\boldsymbol{x^i}$, $\boldsymbol{d_{ij}} = \boldsymbol{x^i} - \boldsymbol{s^j}$ and $d_{ij} = |\boldsymbol{d_{ij}}|$ represents the distance between the i-th collocation point and the j-th source point. Collocating N observation points with respect to N source points we get the linear system of equations

$$\begin{bmatrix} \frac{\partial G_{1,1}^2}{\partial r_1^2} & \cdots & \cdots & \frac{\partial G_{1,N}^2}{\partial r_1^2} \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \frac{\partial G_{N,1}^2}{\partial r_N^2} & \cdots & \cdots & \frac{\partial G_{N,N}^2}{\partial r_N^2} \end{bmatrix} \cdot \begin{bmatrix} \alpha_1 \\ \vdots \\ \vdots \\ \alpha_N \end{bmatrix} = \begin{bmatrix} T_{rr_1} \\ \vdots \\ \vdots \\ T_{rr_N} \end{bmatrix} . \qquad (9)$$

Since the GOCE observations are given efficiently far from the Earth (approximately 250 km above the Earth's surface), the source points can be located directly on the Earth's surface considering its complicated topography. Such a constellation does not generate any singularities. The unknown coefficients $\{\alpha_j\}$ can be determined by solving the linear system of equations (9). Afterwards, the disturbing potential or its first derivatives can be easily evaluated anywhere above the Earth's surface using Eq. (4) or (5). The problem with the singularities appears when computing the gravity field quantities directly on the Earth's surface. In this case it is possible to use two different strategies: (i) to locate source points on the FB, which needs to be shifted below the Earth's surface, or (ii) to apply ideas of SBM that isolate singularities of the fundamental solution at source points on the Earth's surface.

In the first approach a main problem is to determine an optimal position of the FB. As mentioned earlier, this is largely based on experiences. Therefore, in the presented numerical experiments we will step by step change a depth of FBs testing how it influences the resulting MFS solution on the Earth's surface.

In the second approach the ideas of SBM (Chen and Wang 2010) are implemented to overcome singularities of the fundamental solution. Like MFS, SBM also uses the fundamental solution as the basis kernel function of its approximation. In contrast to MFS, the collocation and source points of SBM are coincident and they are all placed on the physical boundary (Fig.1b) avoiding any FB. For the 3D exterior potential problem described in Eqs. (1-3), the SBM interpolation formulation can be expressed as

$$u\left(\boldsymbol{x}^{\boldsymbol{i}}\right)=\sum_{j=1,i\neq j}^{N}\alpha_j G\left(\boldsymbol{x}^{\boldsymbol{i}},\boldsymbol{s}^{\boldsymbol{j}}\right)+\alpha_i u_{ii}\,, \tag{10}$$

$$q\left(\boldsymbol{x}^{\boldsymbol{i}}\right)=\sum_{j=1,i\neq j}^{N}\alpha_j \frac{\partial G\left(\boldsymbol{x}^{\boldsymbol{i}},\boldsymbol{s}^{\boldsymbol{j}}\right)}{\partial \boldsymbol{n}_{\boldsymbol{x}^i}}+\alpha_i q_{ii}\,, \tag{11}$$

where u_{ii} and q_{ii}, named as the origin intensity factors, denote the singular terms $G(\boldsymbol{x}^{\boldsymbol{i}}, \boldsymbol{s}^{\boldsymbol{j}})$ and $\partial G(\boldsymbol{x}^{\boldsymbol{i}}, \boldsymbol{s}^{\boldsymbol{j}})/\partial \boldsymbol{n}$, respectively, i.e., the diagonal elements of the SBM interpolation matrix. These singularities need to be regularized using some special treatment. Applying the regularization technique proposed in (Gu et al. 2012) and omitting details described in this paper, the original singular term q_{ii} for the Neumann boundary equation (11) can be transformed into the regular term

$$q_{ii} = \frac{1}{P_i}\left[1 - \sum_{j=1, i\neq j}^{N} P_j \frac{\partial G\left(\boldsymbol{x^i}, \boldsymbol{s^j}\right)}{\partial \boldsymbol{n}_{s^j}}\right], \tag{12}$$

where P_i, or P_j, is the area of the co-volume surrounding the collocation point $\boldsymbol{x^i}$, or the source point $\boldsymbol{s^j}$, respectively. To evaluate the origin intensity factor u_{ii} for the Dirichlet boundary equation (10), an inverse interpolation technique can be used. Due to the limited extend of this paper, the readers are kindly addressed to (Gu et al. 2012) for more details.

Since the observations from GOCE are sufficiently far from the Earth's surface, the unknown coefficients $\{\alpha_j\}$ can be determined from the linear system of equations (9). Afterwards, the origin intensity factors u_{ii} and q_{ii} need to be determined and finally Eqs. (10-11) can be used to evaluate the disturbing potential or its first derivatives at the source points directly on the Earth's surface. In this way the problem of singularities can be overcome.

4 Numerical experiments

In the numerical experiments we have processed the GOCE measurements from its first 61 days period, i.e., from Oct 1 to Dec 1 2009. In particular, the radial components V_{rr} of the gravity tensor have been transformed to T_{rr} of the disturbing tensor (Fig.2a) using parameters of the GRS-80 normal gravity field. Then the nonlinear diffusion filtering (Čunderlík et al. 2013) have been applied to reduce the noise included in the input data (Fig.2b). In the first experiment the source points have been located directly on the Earth's surface with a resolution of 0.075 deg. It has corresponded to 5,760,002 points (N) regularly distributed over the Earth's surface. To consider the real topography, the vertical position of the source points were generated from the SRTM30_PLUS global topography model (Becker at al. 2009).

To get the linear system of equations (9), the same number of the input observations (collocation points) has been chosen. Their horizontal positions as well as ordering have been adopted from the source points. This has required an interpolation from the original GOCE measurements (firstly filtered by the nonlinear diffusion). To reduce enormous memory requirements for the full matrix in Eq. (9), an iterative approach for the elimination of the far zones' interactions, primarily proposed for the direct BEM (Čunderlík and Mikula 2010), has been applied. This approach together with a parallel implementation using the MPI procedures enables to reach such a high level of the resolution. The large-scale parallel computations were performed on the cluster with 1 TB of the distributed memory. At first, the unknown coefficients $\{\alpha_j\}$ at the source points have been determined solving the linear system of equations (9). From these coefficients, the disturbing potential or its first derivatives can be evaluated at any point above the Earth's surface. Since we have been interested in their values on the Earth's surface, i.e., directly at the source points, the SBM strategy has been used (see Section 3). For this purpose the unknown origin intensity factors u_{ii} and q_{ii} have been determined (see Eq. (12)).

Afterwards, the MFS approach based on the FB has also been used. A depth of the FB has been step by step changing, namely the vertical positions of the source points, while the input GOCE observations have remained the same. For every new position of the FB, new set of the coefficients $\{\alpha_j\}$ has been determined. From these coefficients, the disturbing potential at points on the Earth's surface (with the same positions as in the first experiment) has been evaluated avoiding the problem of singularities.

All particular solutions have been compared with two GGMs developed by SH-based approach, namely, with the GOCO03S satellite-only model up to degree 250 (Mayer-Gürr et al. 2012) and EGM-2008 combined model up to degree 2160 (Pavlis et al. 2012). Graphs in Figure 3 depict how statistical characteristics of the residuals are changing depending on the FB depth. The standard deviation (STD) of residuals is minimal in case of the depth 20 km. The closer to the Earth's surface, the stronger an impact of the singularities becomes and the STD is asymptotically increasing. A special treatment of the singularities by the SBM approach slightly improve the obtained solution, however, an agreement with GOCO03S is worse than in case that the FB depth is from the interval 5 – 30 km. For the FBs deeper than 30 km, the STD is considerably increasing.

On the other side, the overall mean value of residuals is changing minimally. For the FB depths from the interval 0 – 30 km, it changes less than 0.01 m^2s^{-2} (~1 mm) (Fig.3). Considering the mean value over oceans only, it varies within 0.07 m^2s^{-2} (~7 mm). The mean values over oceans also indicates that the W_0 estimates evaluated from the MFS solutions will differ from one based on GOCO03S less than 0.06 m^2s^{-2} and from other computed from EGM-2008 less than 0.1 m^2s^{-2}.

Figure 4 depicts the disturbing potential on the Earth's surface obtained from the MFS solution (the FB depth = 20 km) and from GOCO03S up to degree 250 as well as their comparison. Analogously, Figure 5 shows the first derivatives (the gravity disturbances) for both models and their comparison. Finally, the geopotential on the DTU10 mean sea surface model (Andersen 2010) is evaluated from the obtained MFS solutions (Fig.6). It allows estimating the W_0 values for the selected integration area. Table 1 summarizes our W_0 estimates from the MFS solutions for the different FB depths. These W_0 estimates can be considered independent from ones obtained using the SH-based GGMs. In spite of quite large differences between the MFS solutions and GOCO03S in zones of abrupt changes of the gravity field, e.g. along edges of the lithospheric plates (Fig.4c, Fig.5c, Fig.6), the W_0 estimates differs less than 0.1 m^2s^{-2}.

5 Conclusions

The paper demonstrates that the method of fundamental solutions is an efficient technique for global gravity field modelling. It has an advantage that the approximate solution by MFS satisfies the Laplace equation also in the computational domain with more complicated boundaries. There is no restriction to have spherical (or ellipsoidal) approximation of the Earth's surface like in the SH-based approach. In contrast to BEM, MFS as a mesh-free method does not involve integral evaluations, which make it more efficient. On the other side, to obtain the gravity field quantities, it involves two computational steps. At first the unknown coefficients at the source points need to be determined and then the potential or its derivatives can be evaluated.

The parallel implementation of MFS and the elimination of far zones' interactions allow high-resolution modelling. In all presented numerical experiments the radial components of the gravity tensor are processed and the source points are distributed with the resolution of 0.075 deg. It provides precise global gravity field models that are in a good agreement with the SH-based GGMs, e.g. GOCO03S and EGM-2008. The overall mean values of residuals are smaller than 0.04 m^2s^{-2}. The mean values over oceans do not exceed 0.1 m^2s^{-2}. Hence, the W_0 estimates evaluated from the MFS solutions differ from ones estimated from GOCO03S or EGM-2008 less than 0.1 m^2s^{-2}. Such small differences indicate a feasibility of the W_0 estimates for the realization of WHS.

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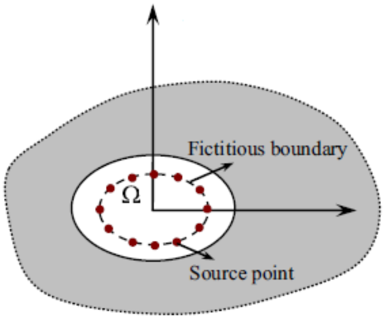
a)



b)

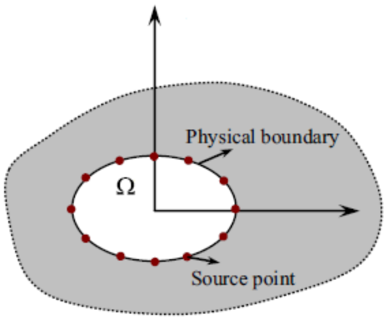
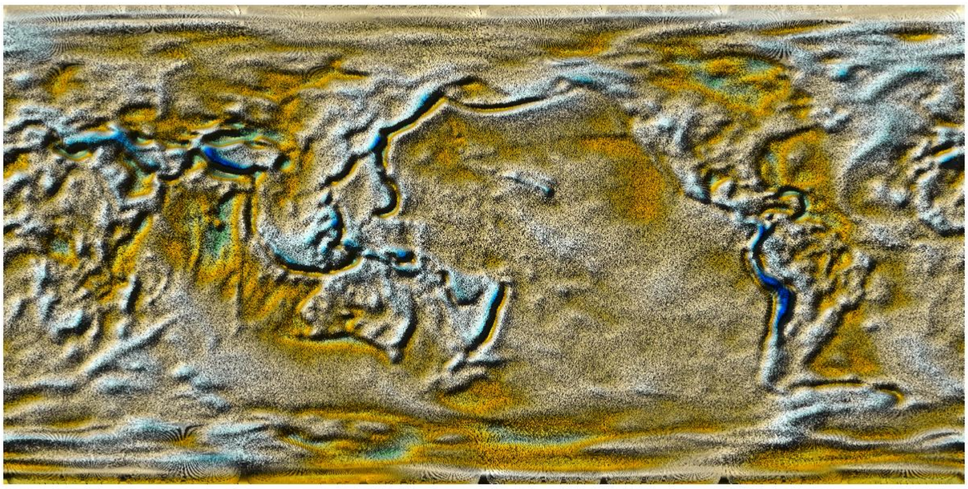


Fig.1: Distribution of the source points for the exterior potential problem using a) the method of fundamental solution (MFS), and b) the singular boundary method (SBM) (from the source: (Gu et al. 2012))

a)



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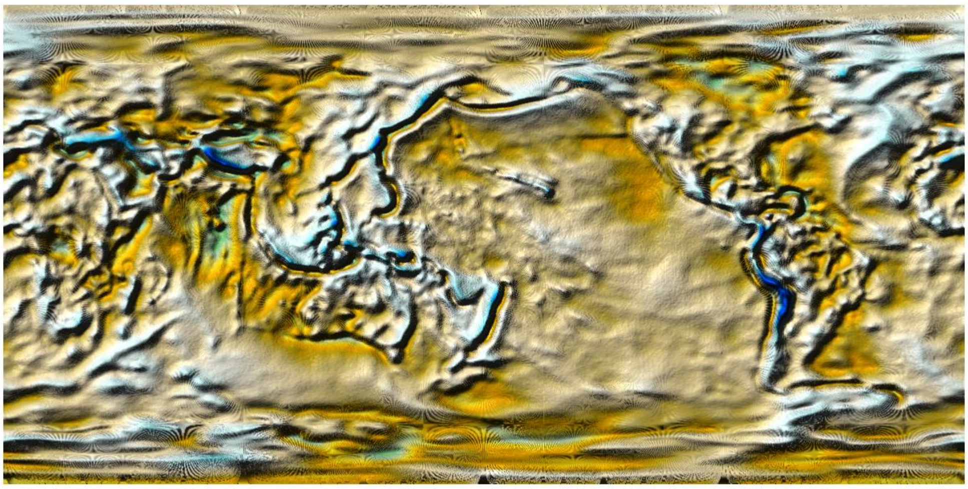


Fig.2: a) The GOCE observations – the radial components T_{rr} of the disturbing tensor, and b) after reducing noise by the nonlinear diffusion filtering

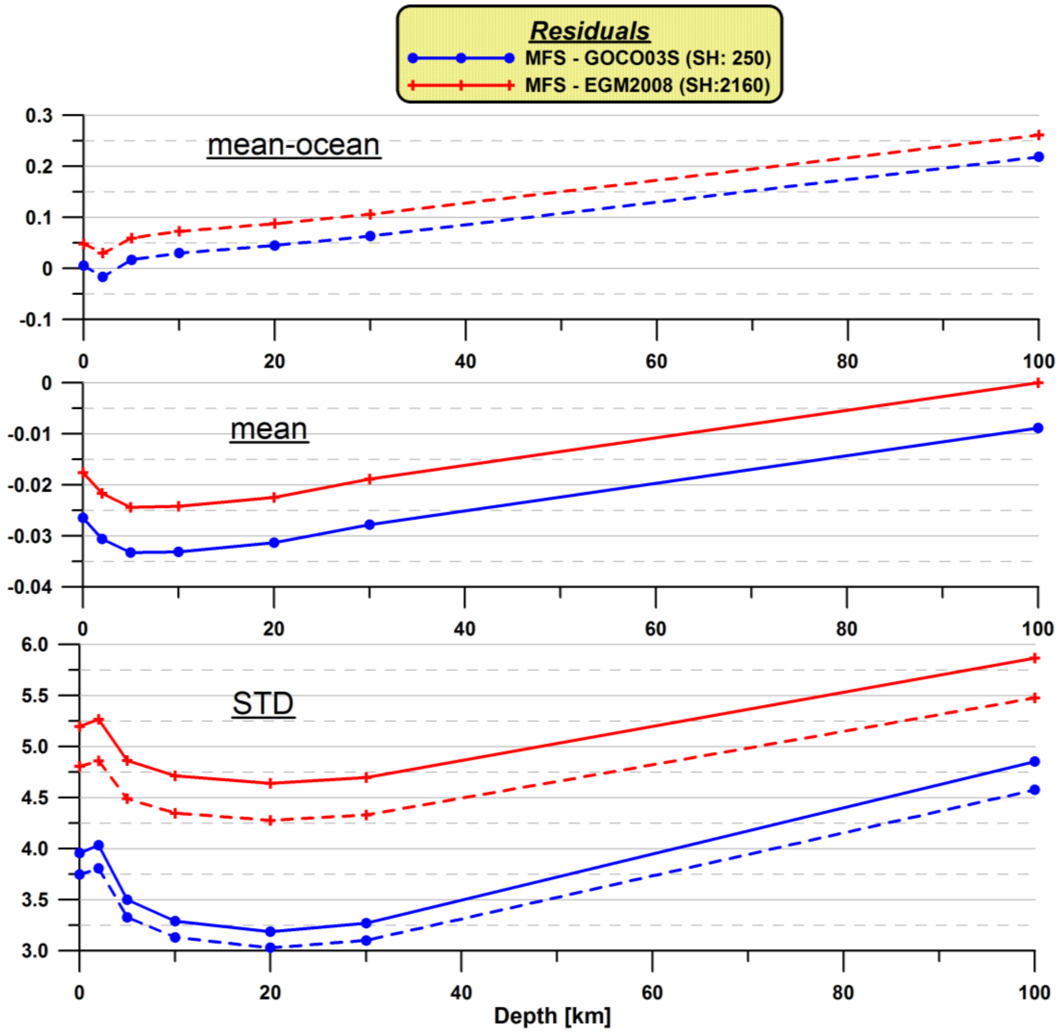
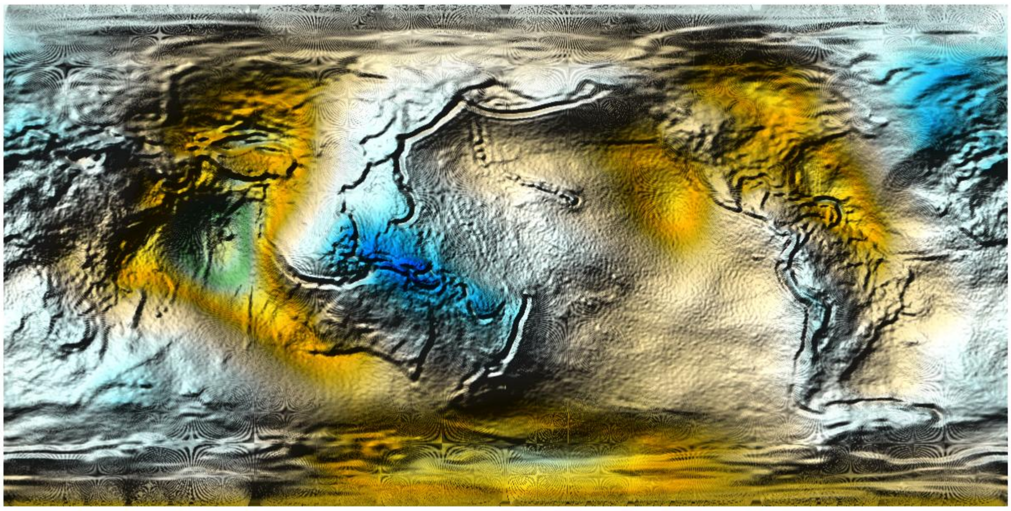
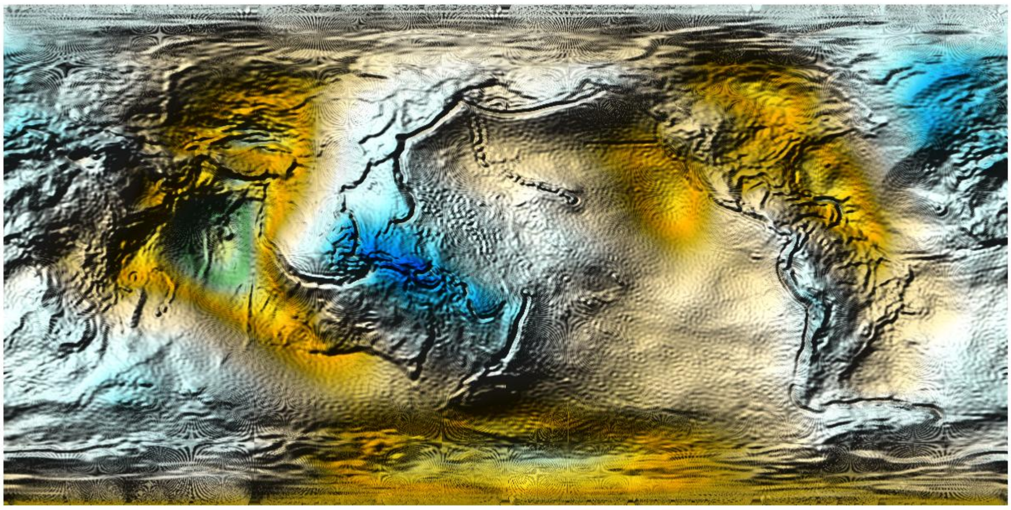


Fig.3: An impact of the depth of the fictitious boundary on the obtained MFS solutions – statistical characteristics of residuals between the MFS solutions and the GOCO03S model up to degree 250 and the EGM-2008 model up to degree 2160

a)



b)



c)

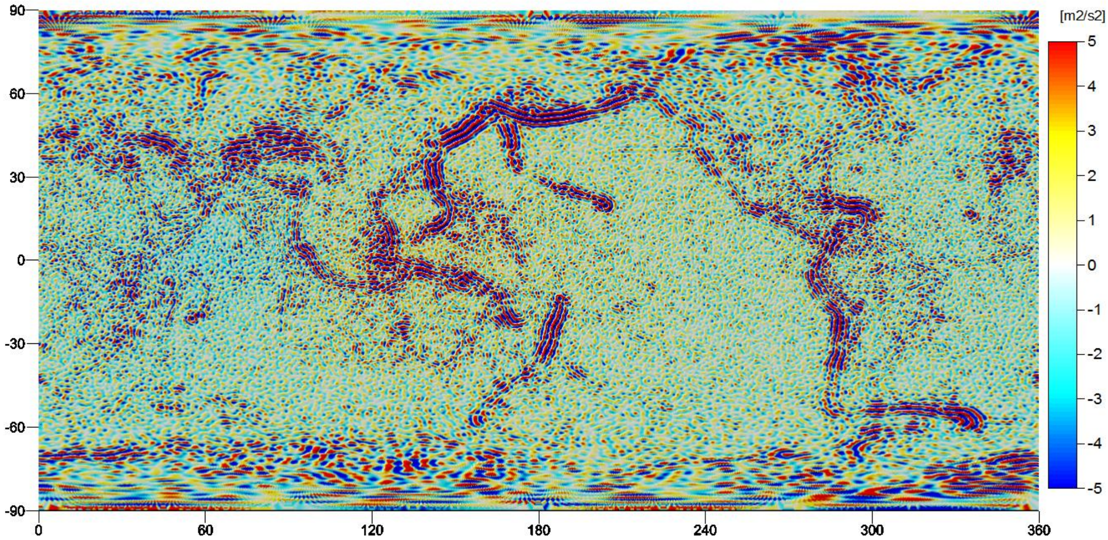


Fig.4: The disturbing potential on the Earth's surface obtained from a) the MFS solution with the fictitious boundary in the depth 20 km, b) from GOCO03S model up to degree 250, and c) the residuals between both models

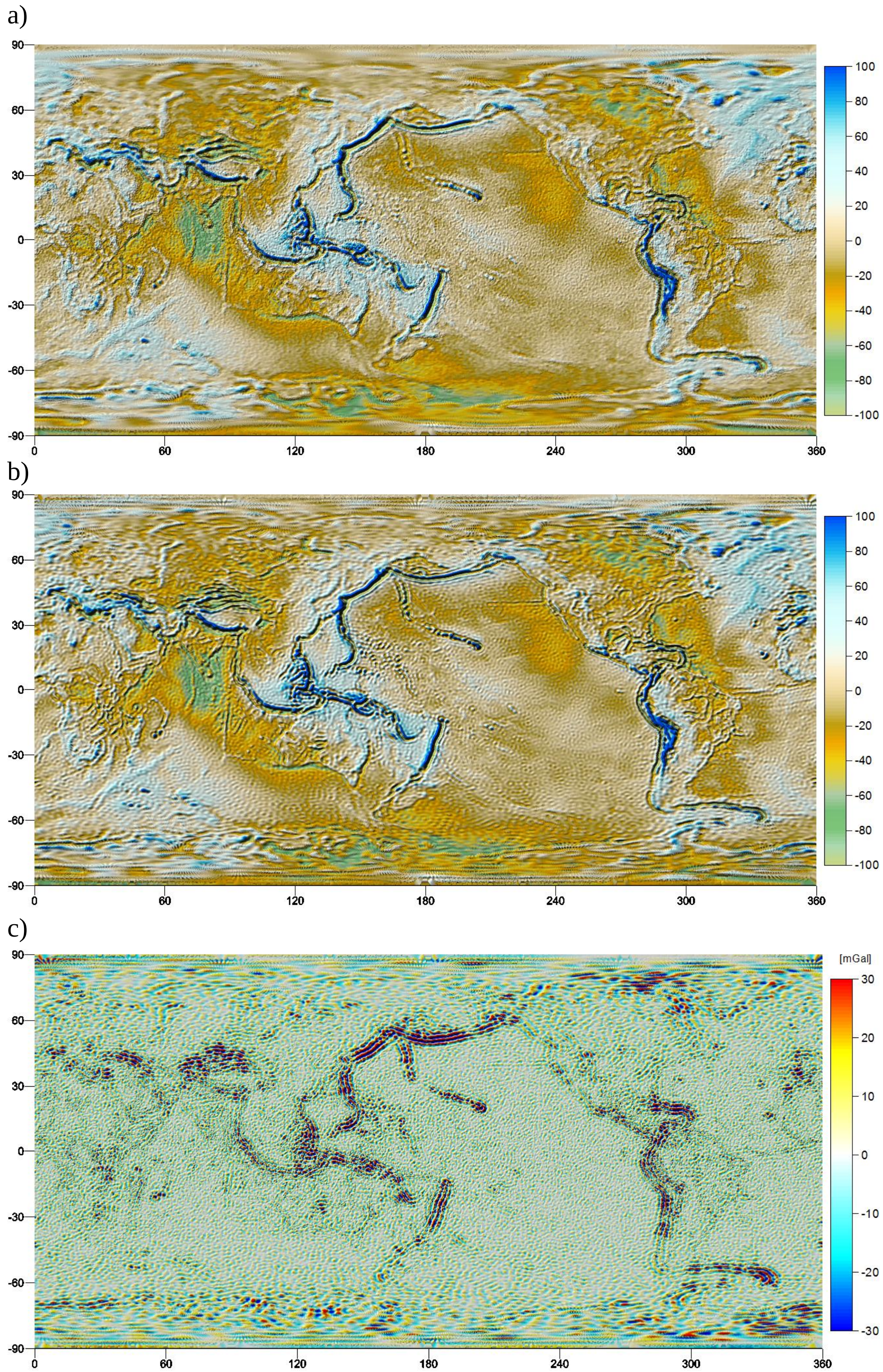


Fig.5: The gravity disturbances on the Earth's surface obtained from a) the MFS solution with the fictitious boundary in the depth 20 km, b) from GOCO03S model up to degree 250, and c) the residuals between both models

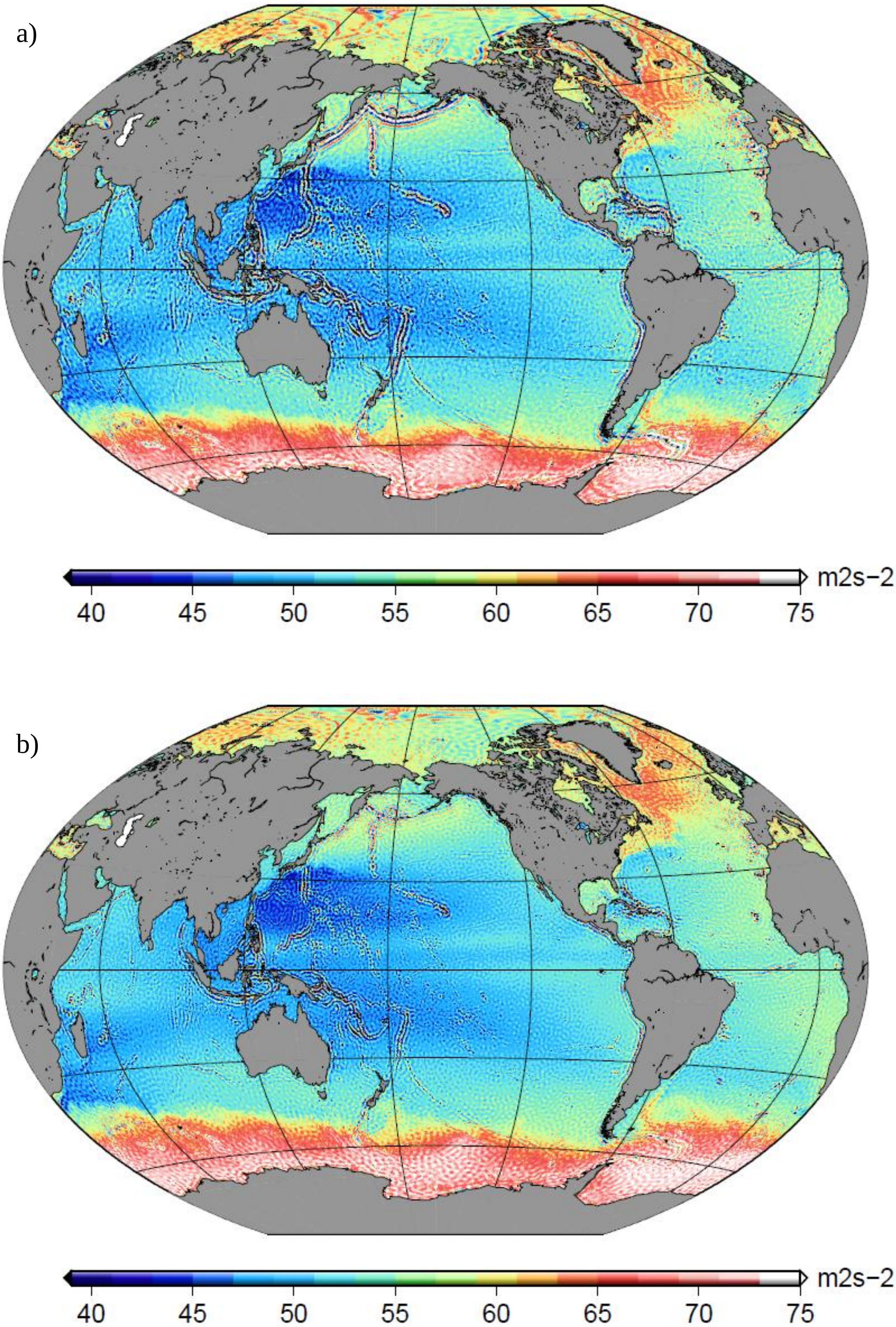


Fig.6: The geopotential on the DTU10 mean sea surface evaluated from a) the MFS solution with the fictitious boundary in the depth 20 km, and b) from GOCO03S model up to degree 250 (the constant 62636800.0 m^2s^{-2} is removed).

Table 1. W_0 estimates evaluated on the DTU10 mean sea surface model (integration area: 82°S – 82°N) from the MFS solutions with the different depths of the fictitious boundaries (FB), and from the GOCO03S and EGM-2008 geopotential models (W_0 units: m^2s^{-2})

FB depth	MFS solution	GOCO03S *(SH up to d/o 250)*	EGM-2008 *(SH up to d/o 2160)*
0 km	62 636 854.01	62 636 854.00	62 636 853.96
2 km	62 636 853.98		
5 km	62 636 854.02		
10 km	62 636 854.03		
20 km	62 636 854.05		
30 km	62 636 854.06		
100 km	62 636 854.22		