SURFACE APPROXIMATIONS AND GEOMETRIC ENERGIES

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Interfacial problems are ubiquitous in physics, biology, mechanics, image processing, computer graphics, etc. Depending on context and applications, there are many different ways to represent an interface, either exact (implicit functions, parametrization, volume boundaries, CAD models, splines) or approximate (meshes, point clouds, phase fields, level sets, pixels/voxels, splines again).

The talk will focus on two mathematical models for interfaces which are well suited for estimating energies of first order (area, perimeter) or second order (curvature-based):

- an explicit model which represents the interface as a measure (more precisely, a varifold) and can describe both continuous (smooth, singular, or diffuse) and discrete surfaces;
- an implicit model, called phase field, which involves an implicit representation of a continuous interface together with a smooth approximation of first and second order energies.

I will discuss the approximation properties of both models, their capacities to encode geometric informations, and the connections between them. I will show a few numerical applications: curvature estimation for point clouds or meshes; volume reconstruction from few slices and application to MRI image reconstruction; mean curvature flow or elastic flows in various contexts (binary/multiphase, isotropic/anisotropic, free/confined), and applications to the simulation of multiphase droplets wetting or nanowire growth.