

# THE H-PRINCIPLE IN FLUID MECHANICS: NON-UNIQUENESS AND ENERGY DISSIPATION

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It is known since the pioneering work of Scheffer and Shnirelman in the 1990s that weak solutions of the incompressible Euler equations behave very differently from classical solutions, in a way that is very difficult to interpret from a physical point of view. Nevertheless, weak solutions in three space dimensions have been studied in connection with a long-standing conjecture of Lars Onsager from 1949 concerning anomalous dissipation and, more generally, because of their possible relevance to Kolmogorov's K41 theory of turbulence.

In joint work with Camillo De Lellis we established a connection between the theory of weak solutions of the Euler equations and the Nash-Kuiper theorem on rough isometric immersions [1]. Through this connection we can interpret the wild behaviour of weak solutions of the Euler equations as an instance of Gromov's celebrated h-principle. In this lecture I will explain this connection, outline the most recent progress concerning Onsager's conjecture [2, 3, 4] and discuss future directions.

## References

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