THE BOUNDEDNESS-BY-ENTROPY METHOD FOR PARABOLIC CROSS-DIFFUSION SYSTEMS

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Cross-diffusion systems describe the dynamic and diffusive interaction of species in multicomponent systems. Examples include multi-species population dynamics, cell biology, ion transport through membranes, and also nonstandard applications like exotic financial derivatives. The systems consist of strongly coupled parabolic equations with a full diffusion matrix. The main challenges are that the diffusion matrix is generally neither symmetric nor positive definite, the equations may be of degenerate type, and standard tools like maximum principles and regularity theory generally do not apply. Major progress has been made by H. Amann end of the 1980s by relating the global-in-time existence of solutions to the validity of certain Sobolev and/or Hölder estimates.

In this talk, recent progress on cross-diffusion systems is reviewed. It has been found that certain systems possess a formal gradient-flow structure inherited from thermodynamical modeling. Formulated in terms of so-called chemical potentials, the new diffusion matrix becomes positive semidefinite, leading to Lyapunov-type estimates for the entropy or free energy. From a physical viewpoint, this expresses the second law of thermodynamics. This idea has been formulated as the boundedness-by-entropy method which allows one to infer natural L^∞ bounds and the global existence of weak solutions simultaneously.

Further novel tools are the duality method of M. Pierre, refined by L. Desvillettes, K. Fellner and others, to derive L^p bounds and generalized Aubin-Lions compactness lemmas by M. Burger, A. Moussa and others. In some cases, even exponential equilibration rates and the uniqueness of weak solutions can be proved by using entropy techniques. Several examples from physics and biology illustrate the strength of these methods.