THE BEAUTY AND EFFICIENCY OF THE GRADIENT DISCRETISATION METHOD

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The gradient discretisation method (GDM) is a generic framework enabling the design and numerical analysis of numerical schemes for elliptic and parabolic PDEs [1]. The GDM is described by the choice of a few discrete elements (a finite dimensional space, a gradient reconstruction, a function reconstruction), together called a "gradient discretisation", which, substituted in the weak formulation of the PDE, give rise to a numerical scheme, the "gradient scheme". Only five properties on the gradient discretisations ensure that the corresponding gradient schemes converge for a wide range of models – linear or non-linear, degenerate, stationary or transient. The GDM encompasses a range of numerical schemes, from finite elements (conforming, non-conforming and mixed) to finite volume methods. The generic GDM convergence analysis ensures that all these schemes converge for all the aforementioned models.

In this talk, I will give an overview of the GDM and of the results it yields. I will in particular focus on two completely novel outcomes of the framework: a uniform-in-time convergence result for numerical approximation of doubly degenerate parabolic equations, and a partial solution of a long-standing super-convergence conjecture for the two-point flux approximation (TPFA) finite volume scheme.

The GDM being based on the continuous weak formulation of the PDEs, most of the talk will be accessible without preliminary knowledge of numerical schemes for elliptic or parabolic problems.

References

[1] Jérôme Droniou, Robert Eymard, Thierry Gallouët, Cindy Guichard, and Raphaèle Herbin. *The gradient discretisation method: A framework for the discretisation and numerical analysis of linear and nonlinear elliptic and parabolic problems.* 2017. Submitted. Available from: https://hal.archives-ouvertes.fr/hal-01382358.